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INITIALIZING ANT (IA) AS AN AGENT IN INITIALIZING POPULATION OF GENETIC ALGORITHM ON FUZZY SHORTEST PATH

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ABSTRACT

Shortest Path (SP) problems in which, the determination of minimal path from source to the destination in the network G=$\{V, E\}$, have many dimensions in various fields of application. The algorithms for Shortest Path (SP) problems have been emerging in higher degree. In real time applications, most parameters (distance, bandwidth, time etc.,) cannot be determined or assigned with the real numbers in solving Shortest Path (SP) problems. It becomes the necessity for the introduction of fuzzy numbers which comprises vertices and edges. Here we consider the generalized trapezoidal fuzzy numbers, which can be dealt with the uncertainty using fuzzy set theory. Genetic Algorithm (GA) provides new space to the emerging algorithm in recent trends of research. In this paper, we concentrate in upgrading population initialization of Genetic Algorithm (GA) using initializing ants resulting in high convergence with ranking of generalized trapezoidal fuzzy numbers, which is proposed recently, as a fitness function. The proposed model is implemented using MATLAB with the test network of 30 nodes and the results reports that the algorithm converges in a more reasonable time in comparison with conventional GA.

KEYWORDS: Genetic Algorithm, Ant Colony, Generalized Trapezoidal Fuzzy Number, Ranking Function, Shortest Path Problem

2010 Mathematics Subject Classification: 03B52, 03E72

1. INTRODUCTION & BACKGROUND

The Shortest Path (SP) problem has accustomed abundant absorption in the literature. Many applications such as communication, robotics, scheduling, transportation and routing, in which, Shortest Path (SP) is applied importantly. While considering a network, the arc length may represent time or cost. Therefore, in real life applications, it can be advised to be a fuzzy set. Fuzzy set theory, proposed by Zadeh [22], is frequently used to accord with uncertainties in a problem.

The fuzzy shortest path problem was first analysed by Dubois and Prade [8]. They utilized the conventional shortest path algorithms, to treat the fuzzy shortest path problem. Klein [14] proposed a dynamic programming recursion-based fuzzy algorithm. Lin and Chen [16] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Okada and Gen [18, 19] presented another algorithm for this problem, using a generalization of Djikstra's algorithm. The algorithm considers the weights of the arcs to be the interval numbers and defined by a partial order of interval numbers.

In order to make evolve the design of fuzzy systems, several metaheuristic learning algorithms are projected. One major improvement class uses evolutionary algorithms (EAs) [21]. These algorithms are heuristic and random. They involve populations of individuals with a particular behavior like a biological development, like crossover or mutation. The most well-known biological process FSs are the genetic FSs [10],[12],[4].[9],[17], that design FSs using Genetic Algorithms (GAs). Another class for FS style is that the Swarm Intelligence (SI) model [15], that could be a comparatively new optimization algorithm compared to EAs. The SI technique studies collective behavior in suburbanized systems. Its development was supported mimicking the social behavior of animals or insects in a shot to seek out the optima in the problem space. Another well-known SI is the ant-colony optimization (ACO) [5].

The ACO technique is impressed by real-ant-colony observations. It is a multiagent approach that was originally projected to resolve troublesome discrete combinatorial- optimization issues, like the traveling salesman problem (TSP) [6], [7]. In some studies, completely different ACO models were applied to FS design problems [11]. In these studies, the antecedent-part parameters of an FS were manually appointed ahead. The consequent-part parameters were optimized in discrete space using ACO. Zainudin Zukhri et. al. [23] proposed Genetic Ant Colony Optimization (GACO) which hybrids Genetic Algorithm (GA) and Ant Colony Optimization (ACO) uses the random selection of chromosome for 1st generation and pheromone.

This paper is organized as follows. In section 2, some basic definitions are reviewed and discussed. Section 3 explains the properties of generalized trapezoidal fuzzy numbers. Section 4 describes the ranking method of generalized trapezoidal fuzzy numbers. Section 5 briefs the network terminology. Section 6 explains the proposed approach of Genetic Algorithm (GA). Section 7 describes the Initializing Ants (IA) used in the population initialization of Genetic Algorithm (GA). In section 8, numerical example along with the example calculation is given. Section 9 deals with the implementation and results. And paper ends with the conclusion and future enhancement in section 10.

2. BASIC DEFINITIONS

The basic definitions of some of the required concepts are reviewed [13] in this section.

2.1 Fuzzy Set

Let X be an universal set of real numbers R, then a fuzzy set is defined as

A= {[x,$\mu_{\tilde{A}}$(x)], x ϵ X}

This is characterized by a membership function: X → [0 1], Where, $\mu_{\tilde{A}}$(x) denotes the degree of membership of the element x to the set A.

2.2 Characteristics of Generalized Trapezoidal Fuzzy Number

A fuzzy set $\tilde{A}$ which is defined on the universal of discourse R, is known to be generalized fuzzy number if its membership function has the following characteristics

a) $\mu_{\tilde{A}} : R \to [0,1] is\ continuous.$
b) $\mu_{\tilde{A}}(x) = 0\ for\ all\ x\ \in (-\infty, a] \cup [d, \infty).$
c) $\mu_{\tilde{A}}(x)\ is\ strictly\ increasing\ on\ [a,b] and\ strictly\ decreasing\ on\ [c,d].$
d) $\mu_{\tilde{A}}(x) = w, for\ all\ x\ \in [b,c], where\ 0 < w \le 1.$

2.3 Membership Function of Generalized Trapezoidal Fuzzy Number

A generalized trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d;w) is known to be a generalized trapezoidal fuzzy number, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{w(x-d)}{(c-d)} & c \le x \le d \end{cases}$$

Let $\tilde{A} = (a, b, c, d; w)$ be a generalized trapezoidal fuzzy number then

a) $R(\tilde{A}) = \frac{w(a+b+c+d)}{4}$, b) mode $\tilde{A} = \frac{w(b+c)}{2}$, c) divergence $(\tilde{A}) = w(d-a)$, d) Left spread $(\tilde{A}) = w(b-a)$, e) Right spread $(\tilde{A}) = w(d-c)$

2.4 Fitness Function

Let $\tilde{A}$= (a_1,b_1,c_1,d_1;w_1) and $\tilde{B}$= (a_2,b_2,c_2,d_2;w_2) be two triangular fuzzy numbers then the addition is defined by $\tilde{A} \oplus \tilde{\boldsymbol{B}}$ = (a_1+a_2, b_1 +b_2, c_1+c_2, d_1+d_2; w_1+w_2)

3. PROPERTIES OF GENERALIZED TRAPEZOIDAL FUZZY NUMBER [2]

Property 3.1 Let $\tilde{A}$ = (a_1 , b_1 , c_1 , d_1 ; w_1) and $\tilde{B}$ = (a_2 , b_2 , c_2 , d_2 ; w_2) be two generalized trapezoidal fuzzy numbers such that

(i) R ($\tilde{A}$) =R ($\tilde{B}$), (ii) mode ($\tilde{A}$) = mode ($\tilde{B}$) and (iii) divergence ($\tilde{A}$) =divergence ($\tilde{B}$) then

a) Left spread ($\tilde{A}$) > Left spread ($\tilde{B}$)iff w_1b_1> w2 b_2

b) Left spread ($\tilde{A}$) < Left spread ($\tilde{B}$)iff w_1b_1< w2 b_2

c) Left spread ($\tilde{A}$) = Left spread ($\tilde{B}$)iff w_1b_1 = w2 b_2

Property 3.2 Let $\tilde{A}$ =(a_1 , b_1 , c_1 , d_1 ; w_1) and $\tilde{B}$ = (a_2 , b_2 , c_2 , d_2 ; w_2) be two generalized trapezoidal fuzzy numbers such that

(i) R ($\tilde{A}$) =R (B), (ii) mode ($\tilde{A}$) = mode ($\tilde{B}$) and (iii) divergence ($\tilde{A}$) = divergence ($\tilde{B}$) then

a) Left spread ($\tilde{A}$) > Left spread ($\tilde{B}$) iff Right spread ($\tilde{A}$) > Right spread ($\tilde{B}$)

b) Left spread ($\tilde{A}$) < Left spread ($\tilde{B}$) iff Right spread ($\tilde{A}$) < Right spread ($\tilde{B}$)

c) Left spread ($\tilde{A}$) = Left spread ($\tilde{B}$) iff Right spread ($\tilde{A}$) = Right spread ($\tilde{B}$)

4. RANKING OF GENERALIZED TRAPEZOIDAL FUZZY NUMBERS

Let $\tilde{A}$ = (a_1 , b_1 , c_1 , d_1 ; w_1) and $\tilde{B}$ = (a_2 , b_2 , c_2 , d_2 ; w_2) be two generalized trapezoidal fuzzy numbers then use the following steps [2] to compare $\tilde{A}$ and B

Step 1: Find R ($\tilde{A}$) and R ($\tilde{B}$)

Case (i) If R ($\tilde{A}$) > R ($\tilde{B}$) then $\tilde{A} > \tilde{B}$

Case (ii) If R ($\tilde{A}$) < R ($\tilde{B}$) then $\tilde{A} < \tilde{B}$

Case (iii) If R ($\tilde{A}$) = R ($\tilde{B}$) then go to step 2.

Step 2: Find mode ($\tilde{A}$) and mode ($\tilde{B}$)

Case (i) If mode ($\tilde{A}$) > mode ($\tilde{B}$) then $\tilde{A} > \tilde{B}$

Case (ii) If mode ($\tilde{A}$) < mode ($\tilde{B}$) then $\tilde{A} < \tilde{B}$

Case (iii) If mode ($\tilde{A}$) = mode ($\tilde{B}$) then go to step 3.

Step 3: Find divergence ($\tilde{A}$) and divergence ($\tilde{B}$)

Case (i) If divergence ($\tilde{A}$) > divergence ($\tilde{B}$) then $\tilde{A} > \tilde{B}$

Case (ii) If divergence ($\tilde{A}$) < divergence ($\tilde{B}$) then $\tilde{A} < \tilde{B}$

Case (iii) If divergence ($\tilde{A}$) = divergence ($\tilde{B}$) then go to step 4.

Step 4: Find Left spread ($\tilde{A}$) and Left spread ($\tilde{B}$)

Case (i) If Left spread ($\tilde{A}$) > Left spread ($\tilde{B}$)

i.e., $w_1b_1>w_2\, b_2$ then $\tilde{A} > \tilde{B}$ (from property 3.1)

Case (ii) If Left spread ($\tilde{A}$) < Left spread ($\tilde{B}$)

i.e., $w_1b_1< w_2\, b_2$ then $\tilde{A} < \tilde{B}$ (from property 3.1)

Case (iii) If Left spread ($\tilde{A}$) = Left spread ($\tilde{B}$)

i.e., $w_1b_1 = w_2\, b_2$ then go to step 5 (from property 3.1)

Step 5 Find w_1 and w_2

Case (i) If $w_1> w_2$ then $\tilde{A} > \tilde{B}$

Case (ii) If $w_1<w_2$ then $\tilde{A} < \tilde{B}$

Case (iii) If $w_1 = w_2$ then $\tilde{A} \sim \tilde{B}$

5. NETWORK TERMINOLOGY

Consider the directed network G (V, E) consisting of a finite set of nodes V={1,2……n} and a set of m directed edges E $\subseteq V \times V$. Each edge is denoted by an ordered pair (i,j) where i, j ∈ v and i≠ j. In this network, we specify two nodes namely source node and the destination node. $\widetilde{d_{ij}}$ denotes the generalized trapezoidal fuzzy number associated with the edge (i,j). The fuzzy distance along the path P is denoted by $\tilde{d}$ (P) =$\sum_{i,j\in P} \widetilde{d_{ij}}$.

6. GENETIC ALGORITHM

Genetic Algorithm (GA) is a type of Evolutionary Algorithm (EA) which is based on the natural selection phenomenon. GA usually has an analogy to the randomness in solving a problem.

6.1 Representation of an Individual (Chromosome)

Each chromosome is represented in binary representation and it is also important which represents the solution in the generations. The representation defines the path traversed and indirectly refers the fuzzy fitness of the chromosome. The number of bits used in representing chromosome is equal to the number of vertices in the network graph G={V, E}. The vertex visited is represented by 1 and 0 represents that the vertex is not visited.

Here, we take 10 vertices network and the representation 1101100001 represents that the path traversed may be 1-2-4-5-10, 1-2-5-4-10, 1-4-2-5-10, 1-4-5-2-10, 1-5-4-2-10 and 1-5-2-4-10 depending on the existence.

6.2 Population Initialization

The initial population is generated randomly in usual GA and each chromosome represents the collection of edges which are represented by generalized trapezoidal fuzzy numbers explained in previous sections. The default population size 20 is used. Initialization of the foremost generation is very important which implicates the convergence rate and existence of chromosomes in population. **Thus we concentrate in upgrading the procedure of initializing population using Initializing Ant (IA) as an agent.**

6.3 Selection Operation

Various selection operations involve Roulette wheel selection, Random selection, Rank selection, Tournament selection and Boltzmann selection [20].Here we choose Rank selection and ranking of generalized trapezoidal fuzzy numbers explained in previous section. As the fuzzy shortest path problem is considered, minimum will survive by ranking the chromosome values. In parent selection phase, two parents with minimal paths are selected using ranking terminology in the generations. Mostly the chromosomes in the generations are arranged in ascending (minimal) order of the rank and hence first two chromosomes can be selected normally.

6.4 Crossover Operation

Crossover operator mates two parent chromosomes and produces children which comprise the essence of two parent chromosome mated. Crossover operation is mainly categorised into two, single point and multi point crossover. The single point crossover has single crossover site whereas multi point crossover has more than single crossover site. There are also some advanced multipoint crossover methods [20] and here we use two point **crossover technique with crossover rate of 0.3.**

6.5 Mutation Operation

The conventional mutation operator performs the minute changes of the reproduced child randomly under a certain rate which undo the degradation of the population due to crossover operation. Here bit flipping is used as mutation operation that carried out using bit complement. Bit complement is nothing but reversing the bits as 1 for 0 and 0 for 1 respectively. The mutation operation is carried out at the rate of 0.1

After the mutation, the obtained chromosome is validated whether the path is continuous and exists in the network. The existed chromosome has sent for the fitness calculation and when the fitness is better than its parents, it will be replaced with parents and used for further generations. The non-continuous chromosomes are discarded.

6.6 Termination Condition

Termination condition produces the optimal solution through the convergence. Mostly termination condition will be the maximum number of generations. Other conditions are the idealness of the chromosomes in the generation. In order to test the algorithm, maximum number of generations can be used as termination condition which clearly represents the convergence of the algorithm.

Here, idealness of the chromosomes is considered as termination condition because of the usage ranking of generalized trapezoidal fuzzy numbers and uncertainty in real numbers. When no change in the optimal fitness (minimal) and the idealness of the chromosomes in generations for at least 5 generations, then the algorithm reaches the termination condition.

Genetic Algorithm

Step 1: Generate the network with vertices and edges with generalized trapezoidal fuzzy numbers

Step 2: Population is initialized using Initializing Ants (IA).

Step 3: Apply cross over process with the parent chromosomes selected using ranking with the rate of 0.3

Step 4: Mutation is carried out with a rate 0.01 randomly and check for the continuity of the path.

Step 5: If continuity fails, then discard the child, else calculate the fitness of child chromosome using definition 2.4

Step 6: When fitness of child is better than parent, child moves for next generation.

Step 7: Repeat the steps 3 to 6 till reaches the termination condition.

Step 8: Report the chromosome with minimal fitness as the solution after termination condition.

7. ANTS AS AGENTS IN INITIALIZING POPULATION IN GENETIC ALGORITHM

Problem Identified in Population Generation

(a) Genetic Algorithm (GA) possesses random selection of chromosomes in initializing the population, in which chromosomes may uncertain in the existence as solution space.

(b) Randomness may also produce discontinuous path as the chromosome in initializing population and leads to unhealthy and unfitted generations.

(c) Since initialization of population implicates the convergence rate, the chromosomes should exist, continuous and provide optimal fitness in solution space.

(d) In order to rectify these problems a new algorithm is constructed with the following assumptions.

Assumptions

- As described in network terminology, n represents the number of vertices in the network and it is assumed that the total possible directed edges in the network will be around $2^{n/2}$. Hence $2^{n/2}$ edges are assumed as optimized number and $2^{n/2}$ Initializing Ants (IA) are considered.

- Since we choose the default population size 20, when $2^{n/2}$ becomes less than 20 (say n=8), it should be assumed as 20 ants as Initializing Ants (IA).
- The property of each IA is assumed to be unique and every IA chooses only the valid vertex for its next visit. Valid vertex is the vertex in which there exists a path between both vertices.
- Here we propose a hybrid ant based algorithm for generating initial population on a network whose edges are assigned by generalized trapezoidal fuzzy numbers with the above mentioned assumptions.

Algorithm for Generating Population

Step 1: Generate the network with vertices and edges with generalized trapezoidal fuzzy numbers

Step 2: $2^{n/2}$ ants are placed in the source vertex.

Step 3: Apply uniqueness for every ant, in selecting the next possible vertex.

Step 4: Continue the traversal till the ant reaches the destination vertex.

Step 5: Calculate the fitness of each path of selected IA using definition 2.4

Step 6: Compare the path address that possess ranking of generalized trapezoidal fuzzy numbers based on rank, mode, divergence, left spread and right spread by definition (Sec 4)

Step 7: Choose 20 continuous existing optimal solutions in ascending (minimal) fitness values.

8. NUMERICAL EXAMPLE

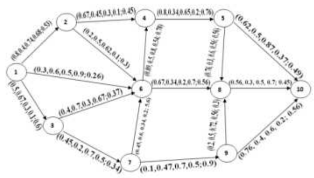

Figure 8.1

We Consider the network G= {V, E} of co vertices (n=10). According to the assumption, we take 2^5 ($2^{10/2}$) IA to serve for the finding of shortest path as initializing population of Genetic Algorithm (GA). Every edge is represented by the generalized trapezoidal fuzzy number. The fitness, ranking and properties of generalized trapezoidal fuzzy number which are described in previous sections, are used to calculate the fuzzy shortest path in a network.

8.1 Selection of IA

Initializing Ant (IA) that has to be used in the algorithm is based on the number of vertices and possible number of edges that is given by $2^{n/2}$. The range of the size of IA is given by $IA\_size = \begin{cases} 2^{n/2}, & n > 8 \\ 20, & otherwise \end{cases}$

Where n represents the number of vertices

Here for the given network, n=10 and number of ant taken = 2^5 ants = 32 ants

8.2 Random Traversal of IA

IAs are assumed to have unique characteristics and the select of path is based on natural selection. The fuzzy distance between the paths is explained in the fitness function of generalized trapezoidal fuzzy number. It is also assumed that the paths travelled by the ants are valid.

Table 7.1: Example Calculation of Fitness for IA (Say a_4)

Present Path	Selection	Fitness
1	6	(0.3,0.6,0.5,0.9;0.26)
1-6	8	(0.97,0.94,0.7,1.6;0.82)
1-6-8	10	(1.53,1.24,1.3,0.86;1.27)
1-6-8-10	-	(1.53,1.24,1.3,0.86;1.27)

During the calculation of fitness, it is necessary to validate the continuity and existence of the path. Consider a IA 1-6-8-10 having chromosomes 1000010101 may represents 1-6-8-10 and 1-8-6-10. While validating, 1-8-6-10 is non continuous and not existed in network and hence discarded. The path 1-6-8-10 is valid under the validation and hence the representation of chromosome 1000010101 is considered to be 1-6-8-10 with the fitness value of (1.53, 1.24, 1.3, 0.86; 1.27).Some of the random paths taken by the ants and its fitness are given below.

8.3 Ranking of Paths Selected By IA

The ranking of generalized trapezoidal fuzzy numbers is used as the selection operation in proposed Genetic Algorithm (GA).Here we demonstrate the calculation of selection operation between two paths using ranking method of generalized trapezoidal fuzzy numbers.

Example

Let $\tilde{A}$ = (0.2, 0.4, 0.6, 0.8; 0.35) and $\tilde{B}$ = (0.1, 0.2, 0.3, 0.4; 0.7)

Step 1

$\tilde{A}$ = 0.175 and $\tilde{B}$ = 0.175. Since $\tilde{A} = \tilde{B}$, so go to step 2

Step 2

Mode ($\tilde{A}$) = 0.175 and mode ($\tilde{B}$) = 0.175. Since mode ($\tilde{A}$) =mode($\tilde{B}$), so go to step 3

Step 3

Divergence ($\tilde{A}$) = 0.21 and divergence ($\tilde{B}$) = 0.21. Since divergence ($\tilde{A}$) = divergence ($\tilde{B}$), so to step 4.

Step 4

Left spread ($\tilde{A}$) = 0.7 and Left spread ($\tilde{B}$) = 0.7. Since Left spread ($\tilde{A}$) = Left spread ($\tilde{B}$), so go to step 5

Step 5

w1 = 0.35, w2 = 0.7 since w1 < w2 $\Rightarrow$ $\tilde{A} < \tilde{B}$

8.4 Population Initialization

Once the behavioural paths of IA are noted, the fitness of the paths is compared using rank selection operation. The best (minimal) 20 paths (population size of GA) are considered as the initial population. The initial population thus produces in the proposed method has the chromosomes with continuous paths and existed in the network.

9. IMPLEMENTATION & RESULTS

The implementation is carried out in Matlab 8.1 (R_{2013a}) 32 bit student version. The assumptions explained before are implemented and the selection of valid path is controlled using adjacency matrix. The network G = {V, E} of 30 nodes with the edges of generalized trapezoidal fuzzy number is initialized. The algorithm is implemented as per the given description and demonstrated numerical calculation. 32,768 IA is separately generated and individual path of IA obtained is measured. The random function with time constraint is used to implement the IA with unique behaviour. The outcome of IA path is comprised and the rank, mode, divergence, left spread and right spread of shortest path is compared using ranking method. The best (minimal) 20 IA (population size of GA) paths are initialized to the GA populations. GA is carried out as per the description.

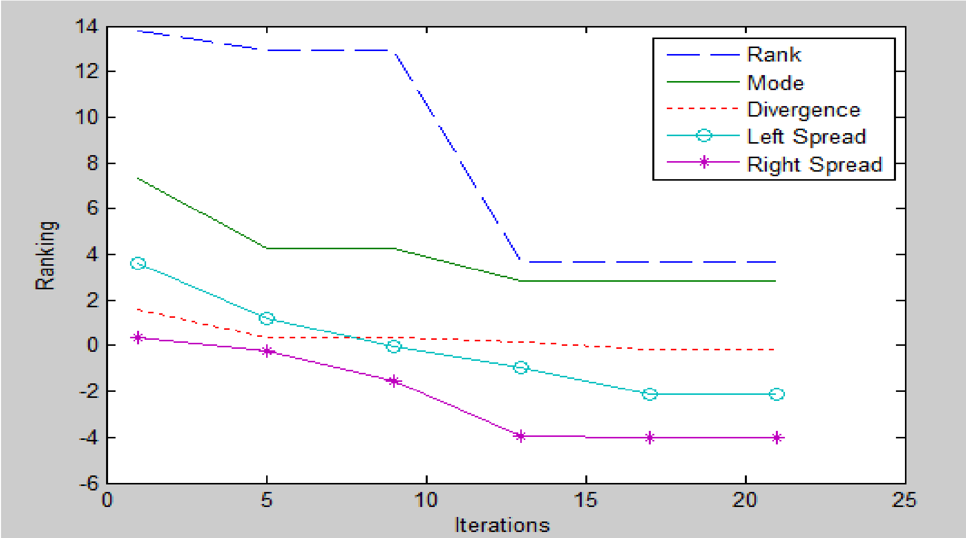

Figure 9.1: Comparison on Ranking Components along Generations

From the figure 9.1**,** it is clear that the ranking method depends on the rank, mode, divergence, left spread and right spread. The path at which all the components attain equilibrium is considered to be the shortest path. Here, generations around **17-21** in which chromosomes possess constant fitness and idealness in the generations for all components of ranking and the path obtained here is considered to be the shortest path.

10. CONCLUSIONS

The Shortest Path (SP) problem in many applications is uncertain in parameters (Distance, Range, etc.). Hence, there occurs the necessity of fuzzy numbers for uncertain parameters. We propose a hybrid ant based optimization algorithm along with the generalized trapezoidal fuzzy numbers and ranking method. The result clears that the proposed Genetic Algorithm (GA) comprises the shortest path with optimal generations using swarm intelligence and all the chromosomes in every generations is continuous and existed in the network. The shortest path obtained for the given network in which all the components attain equilibrium is 1-6-8-10.

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